HW3 Due Feb 10

Midterm 1 Feb 9 Tuesday

Class Time

In person Meet at 209

Sample Test Problems Due Feb 9 midnight

Upload to Moodle.

Email me if you would like to ask questions.

1.9 Proportionality and Power Functions

Def. y is said to be directly proportional to x if there exists a k such that

$$y = kx$$

k is called constant of proportionality.

Example: Force is directly proportional to mass.

$$F = am$$

In this case $k =$ acceleration (a)

Def. y is inversely proportional to x if there exists a k such that

$$y = k\frac{1}{x}$$

In other words, y is directly proportional to $\frac{1}{x}$.

Example: The time taken to travel a given distance is inversely proportional to speed.

$$v = \frac{d}{t}$$

$$\boxed{t = \underbrace{d}_{\text{Constant } k} \cdot \frac{1}{v}}$$

Power functions

$Q(x)$ is a power function of x if $Q(x)$ is proportional to a constant power of x.

$$Q(x) = k \cdot x^{P}$$

Ex. $f(x) = 2x^{7}$

Distinguish from

Exponential functions. $\quad P(x) = P_0 \, a^{x}$

Problem

Which of these are power functions? Write it in the form $y = kx^p$ if possible.

a) $y = \dfrac{5}{x^3}$

Laws of exponents.

- $\dfrac{1}{x^n} = x^{-n}$
- $x^n = \dfrac{1}{x^{-n}}$

Soln. Yes.

$y = 5x^{-3}$

$k = 5$

$p = -3$

b) $y = \dfrac{2}{3x}$

Soln. Yes.

$$\left[\frac{1}{x} = x^{-1} \right]$$

$y = \dfrac{2}{3} \cdot \dfrac{1}{x}$

$= \dfrac{2}{3} \cdot x^{-1}$

$k = 2/3$

$p = -1$

c) $y = \frac{5x^2}{2}$

<u>Soln.</u> Yes

$$y = \frac{5}{2} \cdot x^2$$

$k = \frac{5}{2}$

$p = 2$

d) $y = 5 \cdot 2^x$

<u>Soln.</u> No.

e) $y = 3\sqrt{x}$

<u>Soln.</u> Yes.

$y = 3\sqrt{x}$

$= 3x^{1/2}$

$k = 3$

$p = \frac{1}{2}$

f) $y = (3x^2)^3$

Law of exponent

- $(x^n)^m = x^{nm}$

- $x^n \cdot x^m = x^{n+m}$

Soln. $y = 27(x^2)^3$

$= 27x^6$

$k = 27$

$p = 6$

Alternatively

- $(3x^2)^3$

$= 3x^2 \cdot 3x^2 \cdot 3x^2$

$= 27\, x^2 \cdot x^2 \cdot x^2$

$= 27\, x^6$

Review for Midterm 1

Sample Test 1C

1. An online seller of t-shirts pays \$672 to start up the website and \$6 per shirt, Then sells the t-shirts for \$12 each.

a) Give the cost function.

Soln. Cost = Fixed cost + Variable Cost

$$\boxed{C(q) = 672 + 6q}$$

b) Give revenue function for this situation

Soln. $\boxed{R(q) = 12q}$

c) Give profit function.

$R(q) - C(q)$?
$C(q) - R(q)$?

Soln. Profit $= R(q) - C(q)$

Profit $= 12q - (672 + 6q)$
$= 12q - 672 - 6q$ $= \boxed{6q - 672}$

d. How many t-shirts must be sold in order for the seller to break-even?

Soln. Break even quantity:

Profit $= 0$

$6q - 672 = 0$

$$\frac{\cancel{6}q}{\cancel{6}} = \frac{672}{6}$$

$$\boxed{q = 112}$$

Alternatively:

$R(q) = C(q)$

$12q = 672 + 6q$

$12q - 6q = 672$

$6q = 672$

$q = 112$

$$2-(3+4)$$
$$=2-7$$
$$=-5$$

$$2-3-4$$
$$=2-7$$
$$=-5$$

$$2-3+4$$
$$=-1+4$$
$$=3$$

4. Consider the following function

$$C = 2e^{-0.6r}$$

a) Give the initial value

Soln. 2

b) Give the continuous growth rate as percentage.

Soln. 60% decay

c) Find the annual growth factor

Soln.

$$C = 2\left(e^{-0.6}\right)^r$$

$$= 2\,(0.5488)^r$$

$a =$ growth factor $= 0.5488$

d) Find the annual growth rate.

Soln.

$$a = 1 + r$$

$$0.5488 - 1 = r$$

$$r = -0.4512 = 45.12\%$$ decay

Exponential Functions

variable / input

$$P(t) = P_0 a^t$$

growth factor

Initial value

$a > 1$	exponential growth	(incr.)
$0 < a < 1$	exponential decay	(decr.)

$$\boxed{a = 1 + r}$$

growth factor

growth rate in decimals

Alternative form:

variable / input

$$P(t) = P_0 e^{Kt}$$

continuous growth rate in decimals

Initial value

2.7...

$K > 0$	exponential growth	(incr.)
$K < 0$	exponential decay	(decr.)

$P(t) = P_0 a^t \longrightarrow P(t) = P_0 e^{kt}$

$P(t) = P_0 a^t$

$a = e^k$ for some k

$P(t) = P_0 (e^k)^t$

$= P_0 e^{kt}$

$a = e^k$

$\ln(a) = \ln(e^k)$

$\ln(a) = k$

$\therefore k = \ln(a)$

$P(t) = P_0 a^t \longrightarrow P(t) = P_0 e^{kt}$

$k = \ln(a)$

$P(t) = P_0 a^t$

$P(t) = P_0 e^{kt}$

$P(t) = P_0 e^{kt}$

$= P_0 (e^k)^t$

$= P_0 a^t$

$\boxed{a = e^k}$

5. $f(x) = 3x^7$, give the equation for $g(x)$ such that the graph of $g(x)$ is the graph of $f(x)$ shifted to the left 2 units and shifted up 7 units.

Soln.

$$f(x) \xrightarrow{\text{Shift left}} f(x+2) \xrightarrow{\text{Shift up}} f(x+2)+7$$

$$3x^7 \xrightarrow{\text{Shift left}} 3(x+2)^7 \xrightarrow{\text{Shift up}} \boxed{3(x+2)^7+7}$$

Shifting vertically

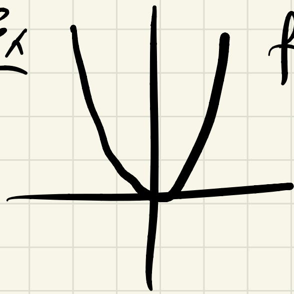
$f(x)$

$f(x) + k$

$k > 0$ shift up

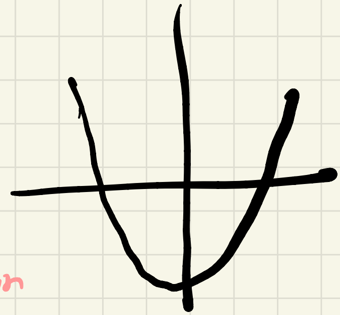
$k < 0$ shift down

Ex

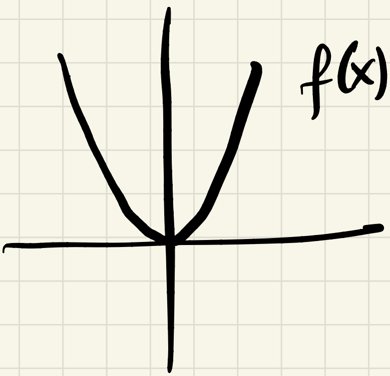


$f(x) = x^2$

shift down

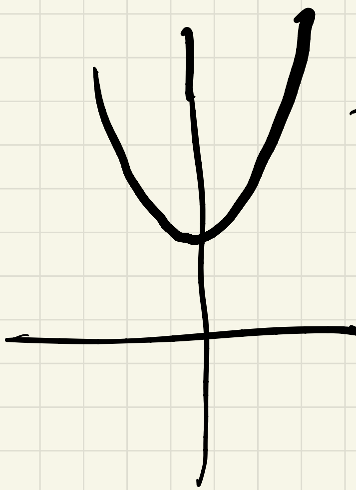


$f(x) - 2$
$= x^2 - 2$



$f(x) = x^2$

shift up



$f(x) + 2$
$= x^2 + 2$

Shifting horizontally

$f(x)$

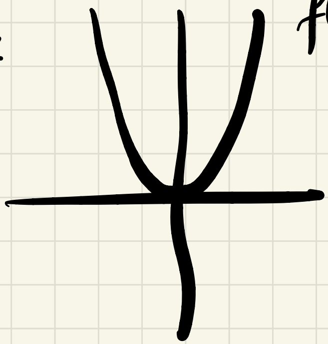
$f(x+k)$

opposite:

$k>0$ shift left

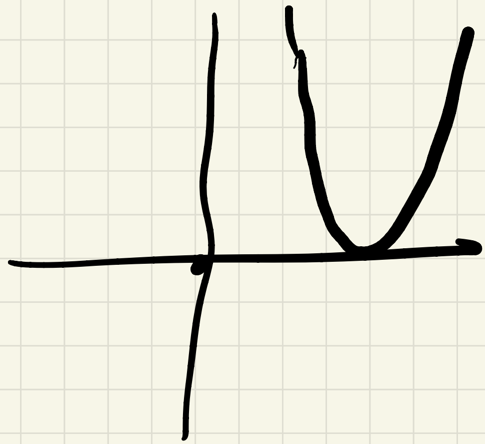
$k<0$ shift right

Ex



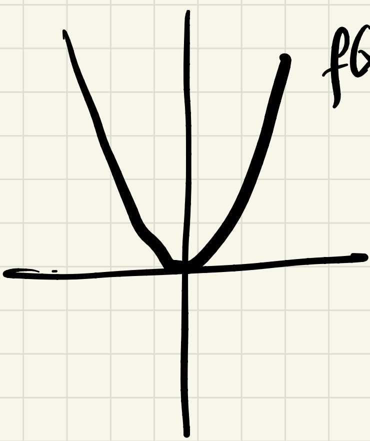
$f(x)=x^2$

shift right



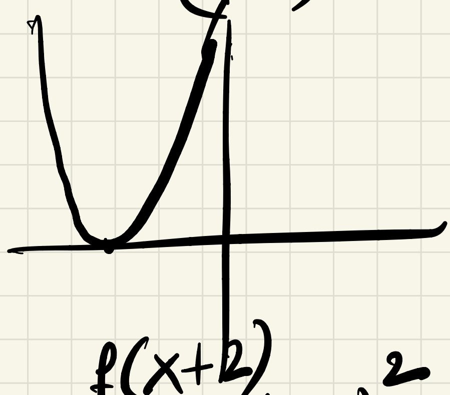
$f(x-2)$

$=(x-2)^2$



$f(x)=x^2$

shift left



$f(x+2)$

$=(x+2)^2$

8. $g(x) = 2x^2 + 16$.

Find the avg. rate of change between -2 and 4.

Soln. Avg. rate of change between $x_1 = -2$ and $x_2 = 4$

$$= \frac{g(x_2) - g(x_1)}{x_2 - x_1}$$

Be careful

$$= \frac{g(4) - g(-2)}{4 - (-2)}$$

$$= \frac{2 \cdot 4^2 + 16 - \left(2(-2)^2 + 16\right)}{4 + 2}$$

$$= \frac{48 - (24)}{6}$$

$$= \frac{24}{6}$$

$$\boxed{= 4}$$

Sample Test 1B

11.

$$90(0.95)^x = 15$$

$$\frac{\cancel{90}(0.95)^x}{\cancel{90}} = \frac{15}{90}$$

$$(0.95)^x = \frac{15}{90}$$

Taking **ln** on both sides:

$$\ln(0.95^x) = \ln\left(\frac{15}{90}\right)$$

$$\frac{x \cdot \cancel{\ln(0.95)}}{\cancel{\ln(0.95)}} = \frac{\ln\left(\frac{15}{90}\right)}{\ln(0.95)}$$

$$x = \frac{\ln\left(\frac{15}{90}\right)}{\ln(0.95)}$$

$$\boxed{= 34.93}$$