Homework 3

Instructions: Write your solutions on paper or a writing tablet, scan it and upload it to canvas. The file must be in pdf extension. Show neat and complete work and make sure that your scan is legible. Label your solutions and make sure they are in increasing order.

1. Are the following vectors/solutions linearly independent or dependent? Show work.
   (a) vectors $y_1 = [3, 4]$ and $y_2 = [5, 7]$
   (b) vectors $y_1 = [2, 18]$ and $y_2 = [\frac{2}{3}, 9]$
   (c) $y_1 = e^x$ and $y_2 = e^{5x}$
   (d) $y_1 = e^x$ and $y_2 = e^{x+7}$
   (e) $y = \sin(x)$ and $y_2 = \cos(x)$
   (f) $y_1 = e^x \sin(x)$ and $y_2 = e^x \cos(x)$
2. Use your knowledge of Taylor series to prove the Euler's formula:
   $$e^{i\theta} = cos(\theta) + isin(\theta)$$
   Remark: This explains why guessing the solution e^{rx} subsumes guessing the solution $sin(x)$ or $cos(x)$ for the second order linear differential equation.
3. Prove the superposition principle for nonhomogeneous equations. Suppose that y_1 is a solution to $Ly_1 = f(x)$ and y_2 is a solution to $Ly_2 = g(x)$. Show that $y = y_1 + y_2$ solves $Ly = f(x) + g(x)$.
4. Find the general solution to $4y'' + 5y' - 2y = 0$.
5. Find all the solutions to $y'' - 5y' + 4y = 0$ for $y(0) = -1$, $y'(0) = 1$.
6. Find the general solution to $y'' - 8y' + 16y = 0$ for $y(0) = 2$, $y'(0) = 1$.

7. Find the general solution to $y'' - 4y' + 4y = 0$.

8. Find the general solution to $y'' + 6y' + 12y = 0$.

9. Find the general solution to $2y'' + y' + 2y = 0$ for $y(0) = 1$, $y'(0) = -2$.