Homework 5

Instructions: Write your solutions on paper or a writing tablet, scan it and upload it to canvas. The file must be in pdf extension. Show neat and complete work and make sure that your scan is legible. Label your solutions and make sure they are in increasing order.

1. Write the following systems as a system of first order differential equations:

   (a)

   $$\begin{aligned} x'' - yx' + y^3y' &= cos(t) \\ y'' &= (xx' + y')^2 - x \end{aligned}$$

   (b)

   $$y''' + y'' + 5y' + 4y = \sin(t)$$

   (c)

   $$x''' = x' + t$$

2. Solve

   $$\begin{aligned} y' &= 2x \\ x' &= x + y \end{aligned}$$

   for $x(0) = -1$, $y(0) = 1$

3. Solve the following by using matrix inverses:

   (a)

   $$\begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(b)

$$\begin{pmatrix} 1 & 3 & 2 \\ 4 & 5 & -1 \\ 2 & 3 & -1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

4. Calculate the following:

   (a)

$$\begin{pmatrix} -1 & 3 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} 0 & 3 \\ 4 & 3 \end{pmatrix}$$

   (b)

$$\begin{pmatrix} -1 & 3 & 3 \\ 4 & -2 & 1 \\ 0 & -1 & 2 \end{pmatrix}^2$$

5. Compute **determinant and inverse** of the following:

   (a)

$$\begin{pmatrix} -1 & 3 & 5 \\ 4 & -2 & 2 \\ 1 & -2 & -1 \end{pmatrix}$$

   (b)

$$\begin{pmatrix} -1 & 3 \\ 2 & -2 \end{pmatrix}$$

6. Write the system

$$\begin{aligned} x_1' &= 2x_1 - 3tx_2 + \sin(t) \\ x_2' &= e^t x_1 + 3x_2 + \cos(t) \end{aligned}$$

   in the form $\vec{x}' = P(t)\vec{x} + f(\vec{t})$ where $P(t)$ is a matrix valued function.

7. (a) Verify that the system $\vec{x'} = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \vec{x}$ has two solutions

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4t} \text{ and } \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-2t}$$

(b) Verify that the two solutions above are independent.

(c) Write down the general solution.

(d) Write down the solution in the form $x_1 =?$ and $x_2 =?$

8. (a) Write the following system in matrix notation.

$$x_1' = 2tx_2 + x_1$$
$$x_2' = e^t x_2$$

(b) Solve the system and write in matrix notation.